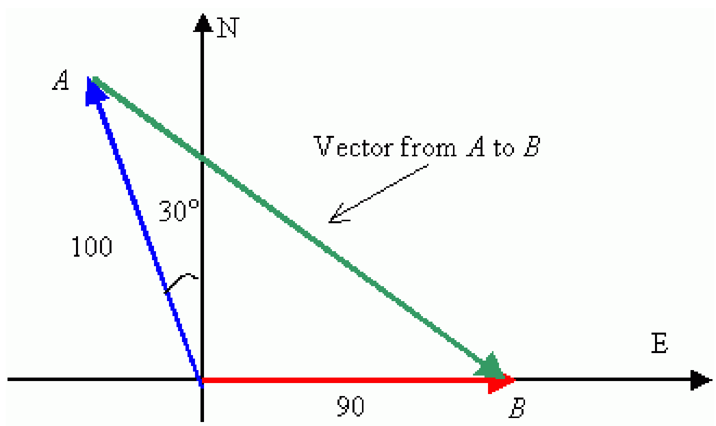
Physics 1100: 2D Kinematics Solutions

Constant Velocity, Distance and Time

1. In a flat desert, two vehicles are initially together but head off at constant velocity in different directions. Vehicle *A* heads off at 50 km/h at 30° west of north while vehicle *B* heads due East at 45 km/h. In two hours, each stops. How far away, and in which direction, is vehicle *B* away from *A*?



Since the velocities are constant the displacement and distance are equal in magnitude. The distance vehicle *A* travels is $D_A = 50$ km/h × 2 h = 100 km. The distance vehicle *B* travels is $D_B = 45$ km/h × 2 h = 90 km. Next we sketch the displacement vectors

Examining the sketch we can see that the vector from A to B is

$$\mathbf{D}_{AB} = \boldsymbol{i}[100 sin(30°) + 90] - \boldsymbol{j}[100 cos(30°)] = \boldsymbol{i}[140] - \boldsymbol{j}[86.60] .$$

Using the Pythagorean Theorem, the distance is

$$D_{AB} = [(140)^2 + (86.60)^2]^{1/2} = 164.6 \text{ km}.$$

The angle can be found using trigonometry,

$$\theta = arctan(86.60/140) = 31.7° .$$

So the direction was 31.7° south of east.

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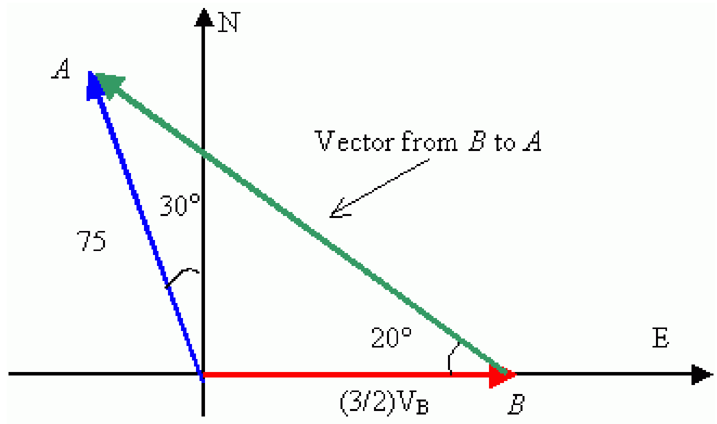
2. In a flat desert, two vehicles are initially together but head off at constant velocity in different directions. Vehicle *A* heads off at 50 km/h at 30° west of north while vehicle *B* heads due East at an unknown speed. Ninety minutes later both stop.

Vehicle *A* now is 190 km away from vehicle *B* at 20° north of west. How fast was *B* moving?



Examining the sketch we can see that the vector from *A* to *B* is

$$\mathbf{D}_{AB} = \boldsymbol{i}[-190\,cos(20°)] + \boldsymbol{j}[190\,sin(20°)] = \boldsymbol{i}[-178.54] + \boldsymbol{j}[64.98]\ .$$

At the same time the x component of $\mathbf{D}_{AB}$ must also equal the x components of the other two vectors. That is

$$178.54 = (3/2)V_B + 75\,sin(30°).$$

Solving for V_B we find, V_B = 94 km/h.

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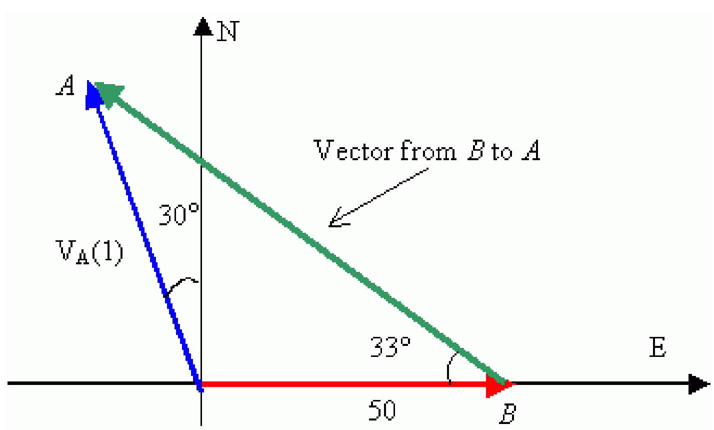
3. In a flat desert, two vehicles are initially together but head off at constant velocity in different directions. Vehicle *A* heads off at unknown speed at 30° west of north while vehicle *B* heads due East at 50 km/h. One hour later both stop. Vehicle *A* now is 95.4 km away from vehicle *B* at 33° north of west. How fast was *A* moving?

   Since the velocities are constant the displacement and distance are equal in magnitude. The distance vehicle *A* travels is $D_A = V_A \times 1$ h $= V_A$. The distance vehicle *B* travels is D_B = 50 km/h × 1 h = 50 km. Next we sketch the displacement vectors



Examining the sketch we can see that the vector from *A* to *B* is

$$\mathbf{D}_{AB} = \boldsymbol{i}[-95.4os(33°)] + \boldsymbol{j}[95.4\,sin(33°)] = \boldsymbol{i}[-80.01] + \boldsymbol{j}[51.96]\ .$$

At the same time the y component of $\mathbf{D}_{AB}$ must also equal the y component of $\mathbf{D}_A$. That is

$$51.96 = V_A cos(30°).$$

Solvingfor V_A we find, V_A = 60 km/h.

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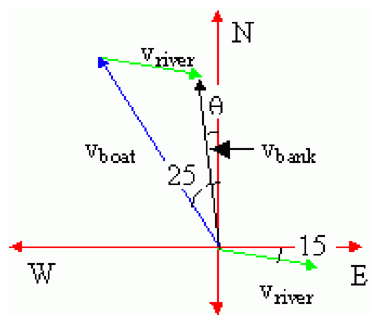
4. A speedboat is travelling at 25 km/h at 25° west of north relative to a river. The river is travelling at 8.0 km/h at 15° south of east relative to the earth. What is the speed of the boat relative to a person on the bank of the river?

The observer on the riverbank sees the combination of the boat's velocity relative to the water plus the velocity of the river relative to the shore where the observer is

$$\mathbf{v}_{bank} = \mathbf{v}_{boat} + \mathbf{v}_{current}.$$

Thus we are dealing with a simple vector addition.



Our given vectors are

$$\mathbf{v}_{boat} = \boldsymbol{i}[-25 sin(25°)] + \boldsymbol{j}[25 cos(25°)] = \boldsymbol{i}[-10.5655] + \boldsymbol{j}[22.6577]$$

and

$$\mathbf{v}_{current} = \boldsymbol{i}[8 cos(15°)] + \boldsymbol{j}[-8 sin(15°)] = \boldsymbol{i}[7.7274] + \boldsymbol{j}[-2.0706] .$$

Thus

$$\mathbf{v}_{bank} = \boldsymbol{i}[-10.5655 + 7.7274] + \boldsymbol{j}[22.6577 + -2.0706] = \boldsymbol{i}[-2.8381] + \boldsymbol{j}[20.5871]$$

Using the Pythagorean Theorem and trigonometry, $v_{bank} = [(-2.8381)^2+(20.5871)^2]^{½}$ = 20.8 km/h and θ = *arctan*(2.8381/20.5871) = 7.8°. So the velocity of the boat relative to the observer is 20.8 km/h at 7.8° west of north.

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5. A captain has his ship moving on the ocean at 12.0 knots at 22.0° west of north. The coast guard office tells him that shore radar indicates he is travelling at 13.2 knots at 28.5° west of north. What is the speed and direction of the ocean current at the ship's location?

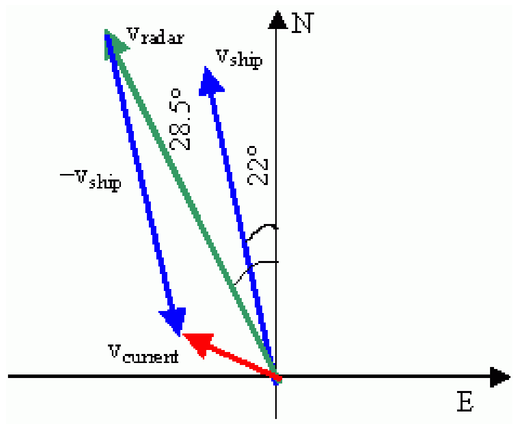
Radar gives the sum of the velocities of the ship and the current

$$\mathbf{v}_{radar} = \mathbf{v}_{ship} + \mathbf{v}_{current} ,$$

so to find the current we need

$$\mathbf{v}_{current} = \mathbf{v}_{radar} - \mathbf{v}_{ship}.$$

Thus to find the current we need to do a simple vector subtraction.



Our given vectors are

$$\mathbf{v}_{radar} = \boldsymbol{i}[-13.2 sin(28.5°)] + \boldsymbol{j}[13.2 cos(28.5°)] = \boldsymbol{i}[-6.2985] + \boldsymbol{j}[11.6004]$$

and

$$\mathbf{v}_{ship} = \boldsymbol{i}[-12 sin(22°)] + \boldsymbol{j}[12 cos(22°)] = \boldsymbol{i}[-4.4953] + \boldsymbol{j}[11.1262] .$$

Thus

$$\mathbf{v}_{current} = \boldsymbol{i}[-6.2985 - -4.4953] + \boldsymbol{j}[11.6004 - 11.1262] = \boldsymbol{i}[-1.8032] + \boldsymbol{j}[0.4742] .$$

We use the Pythagorean Theorem to get the magnitude,

$$v_{current} = [(-1.8032)^2 + (-0.4742)^2]^{½} = 1.86 \text{ knots.}$$

The angle is $\theta = arctan(0.4272/1.8032) = 14.7°$. And thus the direction is 14.7° north of east.

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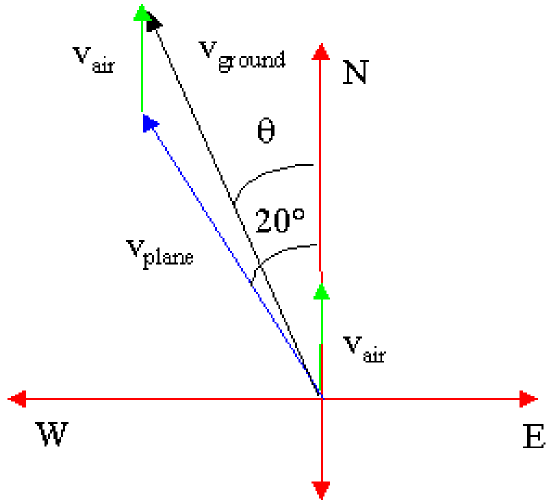
6. An airplane moves with a speed of 420 km/h with respect to the air at an angle of 20.0° west of north. Find the velocity of the plane relative to the ground if the air (i.e. the wind) is moving at:
   a. 25.0 km/h due north.
   b. 17.8 km/h at 37.0° east of south.

The airplane moves relative to the air but the air is moving. An observer on the ground sees the vector sum of the two velocities $\mathbf{v}_{ground} = \mathbf{v}_{plane} + \mathbf{v}_{air}$. So we have a simple vector addition.

(a)



Our given vectors are

$$\mathbf{v}_{\text{plane}} = \boldsymbol{i}[-420\,sin(20°)] + \boldsymbol{j}[420\,cos(20°)] = \boldsymbol{i}[-143.6] + \boldsymbol{j}[394.7]$$

and

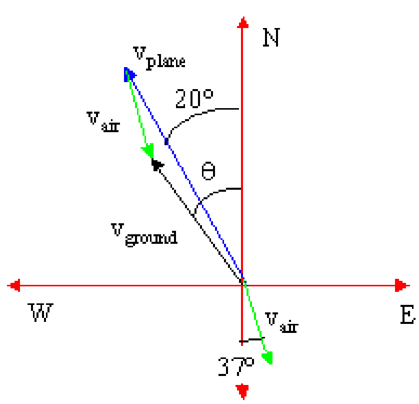
$$\mathbf{v}_{\text{air}} = \boldsymbol{i}[0] + \boldsymbol{j}[25] .$$

Thus

$$\mathbf{v}_{\text{ground}} = \boldsymbol{i}[-143.6 + 0] + \boldsymbol{j}[394.7 + 25] = \boldsymbol{i}[-143.6] + \boldsymbol{j}[419.7] .$$

Using the Pythagorean Theorem and trigonometry, $v_{PG} = [(-143.6)^2+(419.7)^2]^{½} = 443.6$ km/h and θ = *arctan*(143.6/419.7) = 18.9°. So the velocity of the plane relative to the ground is 444 km/h at 18.9° west of north.

(b)



Our given vectors are

$$\mathbf{v}_{\text{plane}} = \boldsymbol{i}[-420\,sin(20°)] + \boldsymbol{j}[420\,cos(20°)] = \boldsymbol{i}[-143.6] + \boldsymbol{j}[394.7]$$

and

$$\mathbf{v}_{\text{air}} = \boldsymbol{i}[0] + \boldsymbol{j}[25] .$$

Thus

$$\mathbf{v}_{\text{ground}} = \boldsymbol{i}[-143.6 + 0] + \boldsymbol{j}[394.7 + 25] = \boldsymbol{i}[-143.6] + \boldsymbol{j}[419.7] .$$

Using the Pythagorean Theorem and trigonometry, $v_{PG} = [(-143.6)^2+(419.7)^2]^{½} = 443.6$ km/h and $\theta = arctan(143.6/419.7) = 18.9°$. So the velocity of the plane relative to the ground is 444 km/h at 18.9° west of north.

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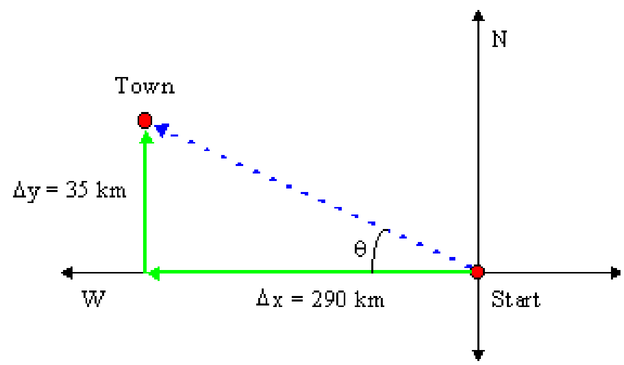
7. An airplane pilot sets a compass course due west and maintains an airspeed of 240 km/h. An hour later, she finds herself over a town which is 290 km west and 35 km north of her starting point.
   a. Find the pilot's actual speed relative to the ground.
   b. Determine the magnitude and direction of the wind's velocity.

Actual ground velocity is the velocity the people in the town would say the pilot had - not the 240 km/h that the pilot saw on her speedometer. Let's start with a sketch of the given information



Since we can assume that the pilot's velocity is constant,

$$V_{Gx} = \Delta x / t = -290 \text{ km} / 1 \text{ h} = -290 \text{ km/h}$$

$$V_{Gy} = \Delta y / t = 35 \text{ km} / 1 \text{ h} = 35 \text{ km/h}$$

So the velocity of the plane is $\mathbf{V}_G = \boldsymbol{i}[-290] + \boldsymbol{j}[35]$.

We convert to polar coordinate form:

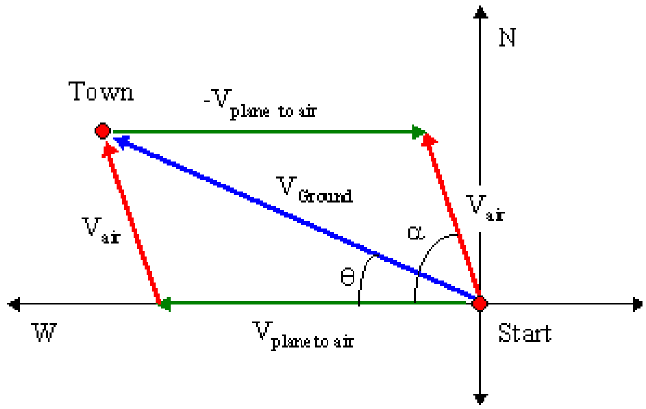
$$V_G = [(V_{Gx})^2 + (V_{Gy})^2]^{½} = [(-290)^2 + (35)^2]^{½} = 292.1 \text{ km/h}$$

$$\theta = arctan(V_{Gy}/V_{Gx}) = arctan(35/290) = 6.88°$$

So the velocity of the plane relative to the ground is 292.1 km/h at 6.88° north of west.

(b) Determine the magnitude and direction of the wind's velocity.

We have the plane's actual velocity to the ground and the plane's velocity relative to the air (the airspeed). Since they are different the air must have added to the motion of the plane, as sketched below.



So we have a little vector subtraction problem, $\mathbf{V}_{air} = \mathbf{V}_G - \mathbf{V}_{PA}$. We break our vectors into components as always and subtract. We already have $\mathbf{V}_G$ and the problem tells us that she heads west at 240 km/h so $\mathbf{V}_{PA} = \boldsymbol{i}[-240] + \boldsymbol{j}[0]$. Thus we find

$$\mathbf{V}_{air} = \boldsymbol{i}[-290 - -240] + \boldsymbol{j}[35 - 0] = \boldsymbol{i}[-50] + \boldsymbol{j}[35] .$$

We convert to polar coordinate form:

$$V_{air} = [(V_{airx})^2 + (V_{airy})^2]^{½} = [(-50)^2 + (35)^2]^{½} = 61.03 \text{ km/h}$$

$$\alpha = arctan(V_{airy}/V_{airx}) = arctan(35/50) = 34.99°$$

So the velocity of the plane relative to the ground is 61.0 km/h at 35.0° north of west.

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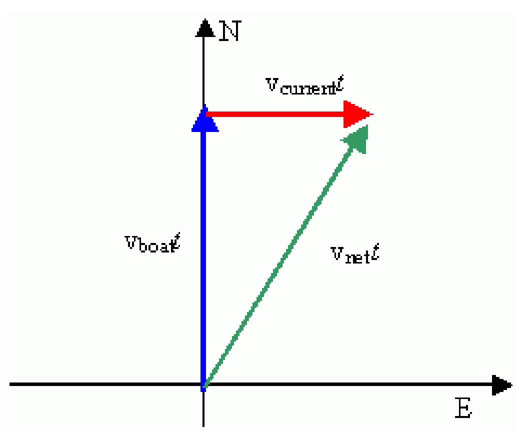
8. A 100-m wide river flows from west to east at 2.00 m/s. A boat moves at 4.00 m/s due north relative to the water. Where on the north bank will the boat land? How long does it take the boat to get there?

The net velocity of the boat is $\mathbf{v}_{net} = \boldsymbol{i}[2] + \boldsymbol{j}[4]$. Since the velocities are constant, the displacement of the boat is given by $\Delta\mathbf{r} = \mathbf{v}_{net}t = \boldsymbol{i}[2t] + \boldsymbol{j}[4t]$, where we don't know how long t it takes to get across. We can sketch the situation



We can also express the landing spot as $\Delta\mathbf{r} = \boldsymbol{i}[d_{unknown}] + \boldsymbol{j}[100]$. But these two expressions must be equal, so $4t = 100$ or it takes 25 seconds to cross and thus $d_{unknown} = 50$ m.

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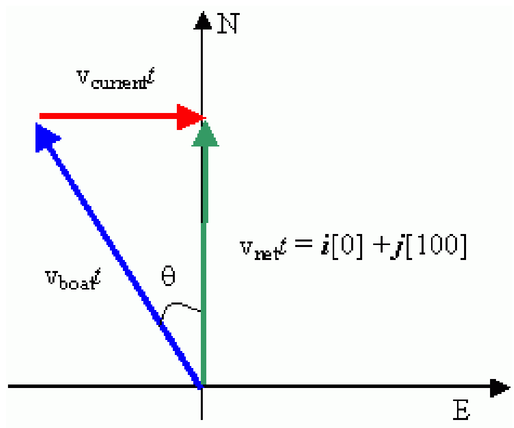
9. A 100-m wide river flows from west to east at 2.00 m/s. A boat moves at 4.00 m/s relative to the water. If a person in a boat wishes to cross from the south bank directly over to the north bank, in which direction must the person head? How long would it take? Where would she land on the north bank?

   Since the velocities are constant we are asking that the net displacement be $\Delta\mathbf{r}_{total} = \boldsymbol{i}[0] + \boldsymbol{j}[100]$. Since the velocities are constant, the displacement of the boat due to the water current is $\Delta\mathbf{r}_{current} = \mathbf{v}_{current}t$ and the displacement of the boat by its own motion is $\Delta\mathbf{r}_{boat} = \mathbf{v}_{boat}t$ where we don't know how long t it takes to get across. A moment's consideration would lead up to expect that the boat must head west of north to counteract the current. The diagram below shows the situation where the unknown angle is labelled θ.



   The next displacement is thus

   $$\Delta\mathbf{r}_{total} = \Delta\mathbf{r}_{boat} + \Delta\mathbf{r}_{current} = \mathbf{v}_{boat}t + \mathbf{v}_{current}t = \boldsymbol{i}[-4sin(\theta)t + 2\ t] + \boldsymbol{j}[4cos(\theta)t].$$

   Comparing this expression for the given displacement the x components give us the equation $-4sin(\theta)t + 2t = 0$. Solving for $sin(\theta)$ we find $sin(\theta) = ½$ or $\theta = 30°$. Using the y component information we find $4cos(\theta)t = 100$ or $t = 100/(4cos(30°)) = 28.9$ seconds.

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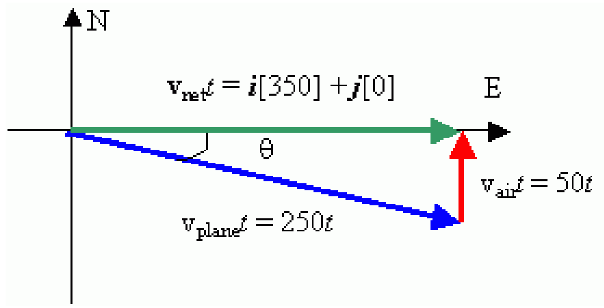
10. A pilot is trying to fly to a town 350 km due east of where he is. A wind is blowing due north at 50 km/h. The plane can fly at 250 km/h with respect to the air. Which way must the pilot head to reach his destination? How long will the trip take?

   Since the velocities are constant we are asking that the net displacement be $\Delta\mathbf{r}_{total} = \boldsymbol{i}[350] + \boldsymbol{j}[0]$. Since the velocities are constant, the displacement of the plane due to the wind current is $\Delta\mathbf{r}_{wind} = \mathbf{v}_{wind}t = \boldsymbol{j}[50t]$. The displacement of the plane by its own motion is $\Delta\mathbf{r}_{plane} = \mathbf{v}_{plane}t$ where we don't know how long t it takes to get to the town. A moment's consideration would lead up to expect that the plane must head south of east to counteract the wind. The diagram below shows the situation where the unknown angle is labelled θ.



The next displacement is thus

$$\Delta\mathbf{r}_{total} = \Delta\mathbf{r}_{plane} + \Delta\mathbf{r}_{air} = \mathbf{v}_{plane}t + \mathbf{v}_{air}t = \boldsymbol{i}[250cos(\theta)t] + \boldsymbol{j}[-250sin(\theta)t + 50t].$$

Comparing this expression with the given displacement, the y components give us the equation $-250sin(\theta)t + 50t = 0$. Solving for $sin(\theta)$ we find $sin(\theta) = 1/5$ or q = 11.5°. Using the x component information we find $250cos(\theta)t = 350$ or $t = 350/(250cos(30°)) = 1.43$ hours.

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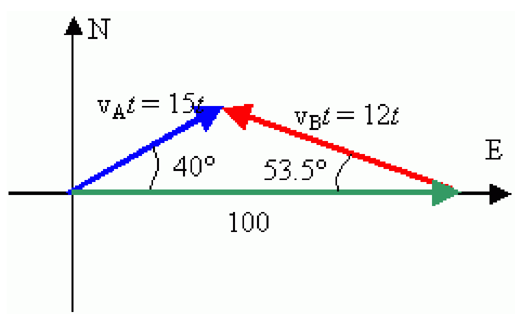
11. Ship A is 100 km due west of ship B. Ship A is travelling at 15.0 km/h at 40.0° north of east. Ship B is travelling at 12 km/h at 53.5° north of west.
    a. Will the two ships collide? How can you tell?
    b. If they do, when will this occur?

Since the velocities are constant the displacements are $\Delta\mathbf{r}_A = \mathbf{v}_A t$ and $\Delta\mathbf{r}_B = \mathbf{v}_B t$. A sketch of the situation, assuming that there is a collision, is shown below.



Careful examination leads us to expect a collision only if the y components of each are the same. Checking $\Delta\mathbf{r}_{Ay} = 15sin(40°)t = 9.64t$ and $\Delta\mathbf{r}_{By} = 12sin(53.5°)t = 9.65t$ which is close enough to suggest they will collide.

To find the time we know that x distance travelled is 100 km, or

$$15cos(40°)t + 12cos(53.5°)t = 100.$$

Solving for t yields, $t = 5.37$ hours.

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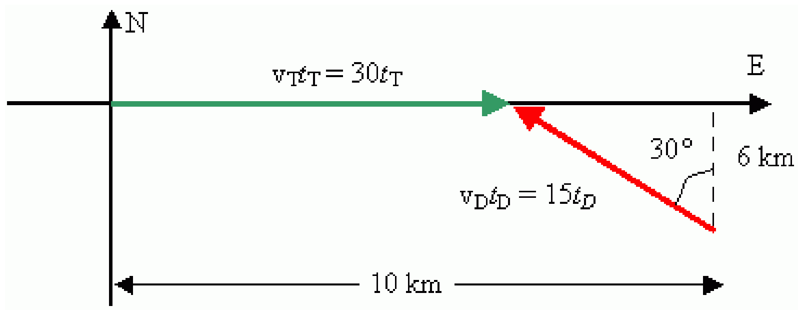
12. A submarine, waiting in ambush, detects an enemy destroyer 10 km east and 6 km south of its position travelling at 15 km/h at 30° west of north. The sub is pointing due east. Its torpedoes travel at 30 km/h. When should it fire its torpedoes to hit

the destroyer?

The destroyer must be due east of the submarine when the torpedoes arrive or they will miss. Now the time that the travelling to that point, t_D, is not the same as the time t_T that the torpedo takes to get to the point of impact. Since the velocities are constant, the displacements are $\Delta \mathbf{r}_D = \mathbf{v}_D t_D$ and $\Delta \mathbf{r}_T = \mathbf{v}_T t_T$. A sketch of the situation, assuming that there is a collision, is shown below.



So first find out how long it takes the destroyer to travel 6 km north. From the sketch, $15t_D cos(30°) = 6$. So it will take $t_D = 0.4619$ h for the destroyer to get to the right location. In the meantime it will have moved $15 sin(30°) * 0.4619$ h = 3.4642 km closer to the sub (i.e. they would be 6.5356 km apart). It takes a torpedo $t_T = 6.5356/30 = 0.2179$ h to cross that distance. So it must fire the torpedoes 0.4619 - 0.21790 = 0.244 h = 14.6 minutes after spotting the destroyer.

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Nonconstant velocity

13. A ball has an initial velocity of 30 m/s at an angle of 30.0° above the horizontal and experiences a constant acceleration of 10.0 m/s^2 at 45.0° above the horizontal. What are the particle's displacement and velocity 5.00 s later?

A 2D kinematics problem is solved by considering it to be two 1D kinematics problems. To do this we consider the ***x*** and ***y*** components separately.

x	***y***
$v_{0x} = 30\ cos(30°)$ $= 25.98$ m/s	$v_{0y} = 30\ sin(30°)$ $= 15.00$ m/s
$a_x = 10\ cos(45°)$ $= 7.071\ m/s^2$	$a_y = 10\ sin(45°)$ $= 7.071\ m/s^2$
t = 5.00s	t = 5.00s
Δx = ?	Δy = ?
v_{fx} = ?	v_{fy} = ?

Considering the ***x*** information, we see that we can use the equation $\Delta x = v_0 t + ½at^2$ to find the displacement. Using the given data,

$$\Delta x = (25.98)(5)+½(7.071)(5)^2 = 218\ m\ .$$

We can use the equation $v_f = v_0 + at$, to find the final x component of velocity,

$$v_f = 25.98 + (7.071)(5.00) = 61.34\ m/s\ .$$

We do the exact same thing with the ***y*** information,

$$\Delta y = (15)(5)+½(7.071)(5)^2 = 163\ m\ \text{, and}$$

$$v_f = 15.00 + (7.071)(5.00) = 50.36\ m/s\ .$$

So the displacement is **D** = (218 m, 163 m). Converting back to polar form, we use the Pythagorean Theorem to get D = $[(218)^2+(163)^2]^{½}$ = 274 m. The angle it makes is given by θ = *arctan*(163/218) = 36.8° . So in polar form, **D** = (274 m, 36.8°).

Similarly, the final velocity $\mathbf{v}_f$ = (61.34 m/s, 50.36 m/s). Converting back to polar form using the Pythagorean Theorem, gives $v_f = [(61.34)^2+(50.36)^2]^{½}$ = 79.4 m/s. The angle it makes is given by θ = *arctan*(50.36/61.34) = 39.4° . So in polar form, $\mathbf{v}_f$ = (79.4 m/s, 39.4°).

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14. A stone has an initial velocity of 15.0 m/s at 22.0° above the horizontal. Fifteen seconds later, it has a velocity of 25.0 m/s at 42.0° above the horizontal. Find the magnitude and direction of the acceleration. Find the displacement in the x and y directions. Assume that the acceleration is constant.

A 2D kinematics problem is solved by considering it to be two 1D kinematics problems. To do this we consider the ***x*** and ***y*** components separately.

	x		***y***
v_{0x}	= 15 *cos*(22°) = 13.908 m/s	v_{0y}	= 15 *sin*(22°) = 5.619 m/s
v_{fx}	= 25 *cos*(42°) = 18.579 m/s	v_{fy}	= 25 *sin*(42°) = 16.728 m/s
t	= 15 s	t	= 15 s
Δx	= ?	Δy	= ?
a_x	= ?	a_y	= ?

Considering the ***x*** information, we see that we can use the equation $\Delta x = ½(v_0 + v_f)t$ to find the displacement. Using the given data,

$$\Delta x = ½(13.908 + 18.579)(15) = 243.7\ m\ .$$

We can use the equation $v_f = v_0 + at$, to find the x component of the acceleration,

$$a_x = (v_f - v_0)/t = (18.579 - 13.908)/15 = 0.3114\ m/s^2\ .$$

We do the exact same thing with the ***y*** information,

$$\Delta y = ½(16.728 + 5.619)(15) = 167.6\ m\ \text{, and}$$

$$a_y = (16.728 - 5.619)/15 = 0.7406\ m/s^2\ .$$

So the displacement is **D** = (243.7 m, 167.6 m). Converting back to polar form, we use the Pythagorean Theorem to get $D = [(243.7)^2+(167.6)^2]^{1/2}$ = 296 m. The angle it makes is given by θ = *arctan*(167.6/243.7) = 34.5° . So in polar form, **D** = (296 m, 34.5°).

Similarly, the acceleration is **a** = (0.3114 m/s^2, 0.7406 m/s^2). Converting back to polar form using the Pythagorean Theorem, gives $a = [(0.3114)^2+(0.7406)^2]^{1/2} = 0.803\ m/s^2$. The angle it makes is given by θ = *arctan*(0.7406/0.3114) = 67.2° . So in polar form, **a** = (0.803 m/s^2, 67.2°).

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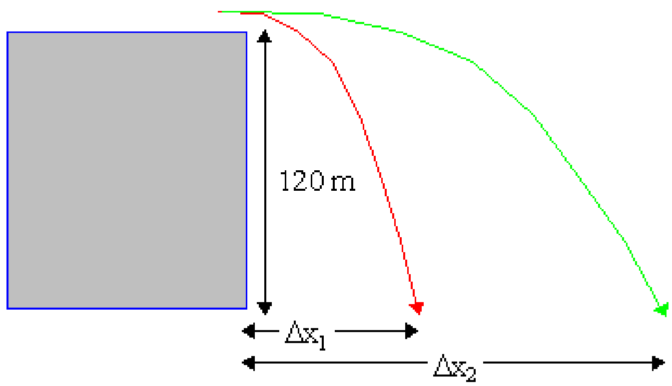
Projectiles

15. A muscular boy and a puny boy of equal height are standing on a cliff 120 m above the ocean. Both throw a rock horizontally out off the cliff at the same time. The muscular boy can throw a rock four times as far as the puny boy. Which boy's rock hits the water first? How long is each boy's rock in the air? If the puny boy's rock hits the water 15.0 m from the base of the cliff, how fast does the muscular boy throw his rock? What is the magnitude and direction of the velocity of each boy's rock just before it hits the water? Sketch the motion.

We will start with a sketch of the problem.



A 2D kinematics problem is solved by considering it to be two 1D kinematics problems. To do this we consider the ***x*** and ***y*** components for each boy separately. The fact that the boys throw the rocks horizontally tell us that the initial y component of velocity is zero. We are told that $\Delta x_2 = 4\Delta x_1 = 60$ m. As well since we are given the height of the cliff $\Delta y_1 = \Delta y_2 = -120$ m. For projectiles, $a_x = 0$ and a = 0 and $a_y = -g$.

	puny boy		*muscular boy*
x	***y***	***x***	***y***
$v_{P0x} = ?$	$v_{P0y} = 0$	$v_{M0x} = v_2$	$v_{0y} = 0$
$v_{Pfx} = ?$	$v_{Pfy} = ?$	$v_{Mx} = ?$	$v_{fy} = ?$
$a_x = 0$	$a_y = -g$	$a_x = 0$	$a_y = -g$
$\Delta x_1 = 15$ m	$\Delta y = -120$ m	$\Delta x_2 = 60$ m	$\Delta y = -120$ m
	t_1		t_2

Examining the y columns, we see that we have enough information to use the equation $\Delta y = v_{0y}t - \frac{1}{2}gt^2$. Notice that the y information is the same for both boys, so the time in the are will be identical. Since $v_{0y} = 0$, we get

$$t = [2\Delta y/g]^{1/2} = [2(-120)/(-9.91)]^{1/2} = <\pm 4.95\ s.$$

We want the forward in time or positive solution, so both rocks are in the air for 4.95 s.

Now that we have the time, we have enough information to find v_{0x} for each boy using $\Delta x = v_{0x}t + \frac{1}{2}a_x t^2$. Since $a_x = 0$,

$$v_{P0x} = \Delta x_1 / t = (15\ m)/(4.95\ s) = 3.03\ m/s,$$

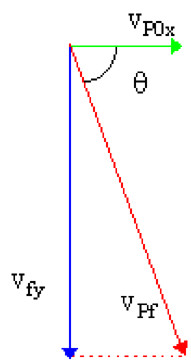
$$v_{M0x} = \Delta x_2 / t = (60\ m)/(4.95\ s) = 12.12\ m/s.$$

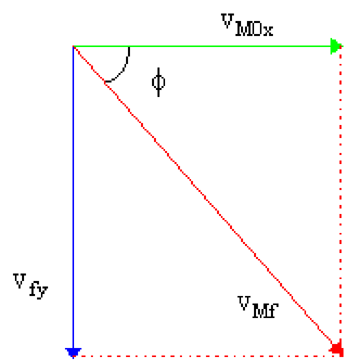
So the muscular boy throws the rock four times as fast as the puny boy.

To find v_f we need to find the components of the final velocity, v_{fx} and v_{fy}, first. Since $a_x = 0$, $v_{Pfx} = v_{P0x} = 3.03$ m/s and $v_{Mfx} = v_{M0x} = 12.12$ m/s. We can use $v_{fy} = v_{0y} + a_y t$ to determine the y components. Since $v_{0y} = 0$ and $a_y = -g$ in both cases,

$$v_{Mfy} = v_{Pfy} = -(9.81)(4.95) = -48.52\ m/s\ .$$

We can use the Pythagorean Theorem and trigonometry to find v_f in each case.





puny boy

$v_{Pf} = [(3.03)^2+(-48.52)^2]^{1/2}$
$= 48.6$ m/s

$\theta = arctan(48.52/3.03)$
$= 86.4°$

muscular boy

$v_{Pf} = [(12.12)^2+(-48.52)^2]^{1/2}$
$= 50.0$ m/s

$\varphi = arctan(48.52/12.12)$
$= 75.9°$

So the final velocity of the puny boy's rock is 48.6 m/s at 86.4° below the horizontal, while the muscular boys rock has a final velocity of 50.0 m/s at 75.9° below horizontal.

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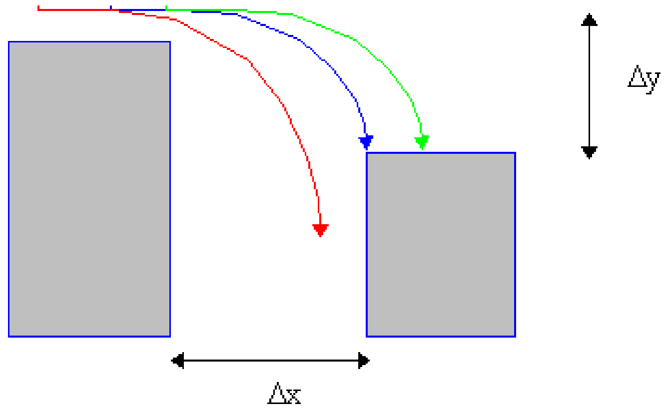
16. You are trapped on the top of a burning building. Death is imminent and help is nowhere in sight. There is a safe building 6.50 m away and 3.00 m lower. You decide to try and make it across. You run horizontally off your building at 8.10 m/s. Do you make it across? If you don't, how much faster must you be going?

First we sketch the situation and possible outcomes.



While you are jumping, you are a projectile. We solve projectile motion problems by considering the x and y components separately, keeping in mind that the time in air is common. We write out the ***i*** and ***j*** information in separate columns including the information that we can infer or that we are supposed to know like the fact that running horizontally implies that $v_{oy} = 0$..

i		***j***	
$\Delta x_{safe} = 6.50$ m		$\Delta y_{safe} = -3.00$ m	minus indicates down
$a_x = 0$	No x component for projectiles	$a_y = -g = -9.81$ m/s^2	gravity acts down
$v_{0x} = 8.10$ m/s		$v_{0y} = 0$ m/s	horizontal takeoff means no vertical component
$t_{air} = ?$	common	$t_{air} = ?$	

Looking at the x information, we see that we have enough data to find t_{air}. The kinematics equation that has all four quantities is $\Delta x = v_{0x}t + ½a_xt^2$. Since $a_x = 0$ for a projectile, this equation becomes $\Delta x = v_{0x}t$. Solving for t, we get

$$t = \Delta x / v_{0x} = (6.50 \text{ m}) / (8.10 \text{ m/s}) = 0.8025 \text{ s} .$$

This is the time it would take you to cross a horizontal distance of 6.50 m. You must be in the air for at least this long if you are to safely make it across to the next building.

On the other hand, looking at the y information, we see that we also have enough data to find t_{air}. The kinematics equation that has all four quantities is $\Delta y = v_{0y}t + ½a_yt^2$. We know $v_{0y} = 0$ since you ran off the roof horizontally, and since $a_y = -g$, this equation becomes $\Delta y = -½gt^2$. Solving for t, we get

$$t = \{ -2\Delta y / g \}^{½} = \{(-2 \times -3.00 \text{ m}) / (-9.81 \text{ m/s}) \}^{½} = 0.7821 \text{ s} .$$

This is the time it takes you to fall a vertical distance of 3.00 m. If you do reach the other building, then this is how long you were in the air.

Since the time it takes to cross the horizontal distance is less than the time you have, you have don't make it across.

To make it across safely you of course would need to run off the roof faster. Since $a_x = 0$ for projectiles, this equation become $\Delta x = v_{0x}t$ where t is now the 0.7821 s. Solving for v_{0x}, we get ,

$$v_{0x} = \Delta x / t = (6.50 \text{ m}) / (0.7821 \text{ s}) = 8.31 \text{ m/s} .$$

If you were able to run at 8.31 m/s you would safely make it to the other building.

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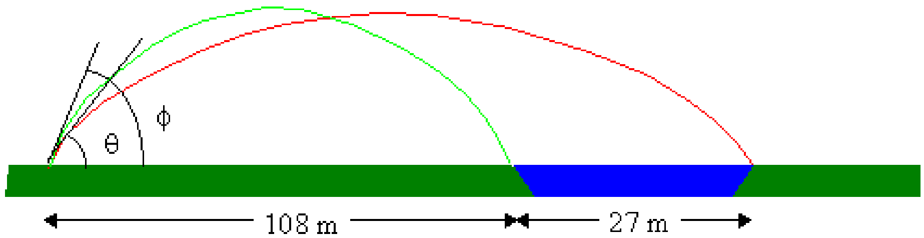
17. A golfer can give a golfball an initial speed of 40 m/s. The golfer can also vary the angle at which it leaves the tee. There is a 27–m wide pool of water 108 m away along the direction of the golfer's swing. What two angles can he hit the ball and have it land just in front of the pool? What two angles can he hit the ball and have it land just past the pool? What range of angles must be avoided to keep the golfball out of the water? Assume the balls don't roll after they hit the ground. The identity $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ will be useful.

A good sketch helps us to understand the problem.



We see that if the angle is between θ and φ , the golfball lands in the pond.

We will consider this problem as two separate level–to–level projectile problems.

short		long	
x	***y***	***x***	***y***
$v_{0x} = v_0\cos\theta$	$v_{0y} = v_0\sin\theta$	$v_{0x} = v_0\cos\varphi$	$v_{0y} = v_0\sin\varphi$
$\Delta x = 108$ m	$\Delta y = 0$	$\Delta x = 108+27$ m $= 135$ m	$\Delta y = 0$
$a_x = 0$	$a_y = -g$	$a_x = 0$	$a_y = -g$
$t = ?$		$t = ?$	

Considering the golfball that lands just short of the pool, we see we have two unknowns θ and t. We will need to write an equation for each column and solve the two equations together;

$$\Delta x = (v_0\cos\theta)t , \qquad (1)$$

$$0 = (v_0\sin\theta)t - \tfrac{1}{2}gt^2. \qquad (2)$$

In equation (2), we can divide by t, so the equation simplifies to

$$t = (2v_0\sin\theta)/g .$$

This can be substituted into equation (1) and we get

$$\Delta x = 2(v_0)^2\cos\theta \sin\theta /g. \qquad (3)$$

This is difficult to solve unless we recall the identity

$$2 \cos\theta \sin\theta = \sin(2\theta) .$$

Using the identity, equation (3) becomes

$$\sin(2\theta) = g\Delta x/(v_0)^2. \qquad (4)$$

So inverting the (4) yields

$$\theta = \tfrac{1}{2}\mathit{arcsin}[g\Delta x/(v_0)^2].$$

For $\Delta x = 108$ m, this yields $\theta = \tfrac{1}{2}(41.47°) = 20.7°$ but this is not the only answer since the arcsine functions give multiple solutions. There are is another positive solutions $\theta = \tfrac{1}{2}[180° - 41.47°] = 69.3°$. Note that the first and second solution add to 90° . So the golfer can hit the ball at either 20.7° or 69.3° and just avoid the pool.

For the long shot, our calculations are identical. For $\Delta x = 135$ m, $\varphi = 27.9°$ or 62.1° . So the golfer can hit the ball at either 27.9° or 62.1° and have it land just beyond the pool.

So avoid the watertrap, the angles to avoid are $20.7° < \alpha < 27.9°$ and $62.1° < \beta < 69.3°$.

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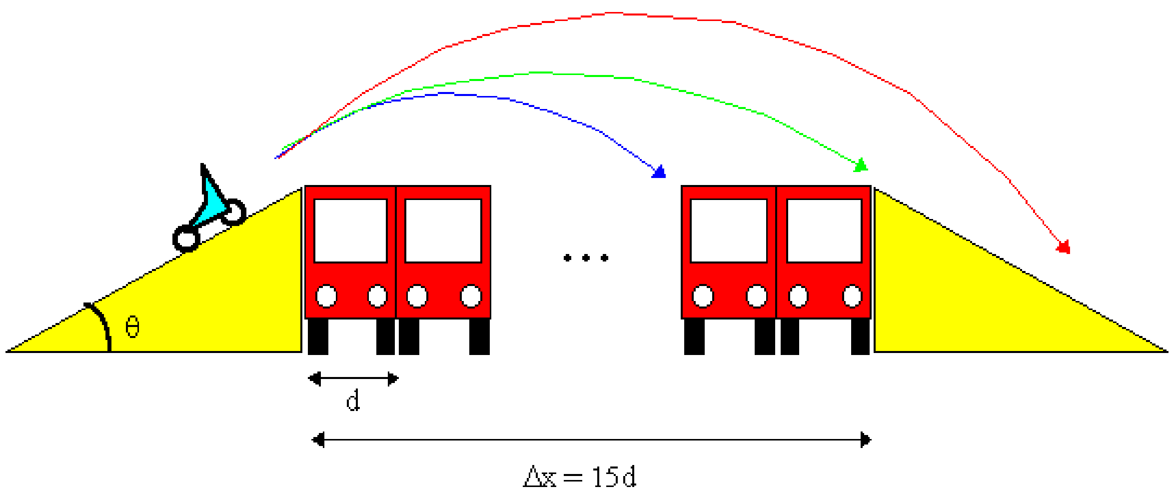
18. A stunt motorcyclist is trying to jump over fifteen buses set side to side. Each bus is 2.50 m wide and a 30.0° ramp has been installed on either side of the line of buses. What is the minimum speed at which she must travel to safely reach the other side? How long will she be in the air?

First we sketch the situation and possible outcomes.



While the motorcyclist is jumping, she is a projectile. We solve projectile motion problems by considering the x and y components separately, keeping in mind that the time in air is common. We write out the ***i*** and ***j*** information in separate columns including the information that we can infer or that we are supposed to know. We see from the sketch that if the motorcyclist is successful, then this is an example of level–to–level flight and $\Delta y = 0$. Note that the initial velocity is broken into components.

i		***j***	
$\Delta x_{safe} = 15 \times 2.50$ m $= 37.5$ m		$\Delta y_{safe} = 0$	level to level flight
$a_x = 0$	No x component for projectiles	$a_y = -g = -9.81$ m/s^2	gravity acts down
$v_{0x} = v_0\, cos\theta$	v_0 is unknown	$v_{0y} = v_0 sin\theta$	
$t_{air} = ?$	common	$t_{air} = ?$	common

Looking at the x and y information, we see that we have two unknowns, v_0 and t, for both. While we cannot solve any equation for x or y since there are two unknowns, both can be solved together. The appropriate kinematics equation that has all four quantities for x and for y is:

$$\Delta x = v_{0x}t + \tfrac{1}{2}a_x t^2, \text{ and } \Delta y = v_{0y}t + \tfrac{1}{2}a_y t^2.$$

We substitute in known quantities to get

$$\Delta x = v_0 \cos\theta \, t \qquad (1),$$

and

$$0 = v_0 \sin\theta \, t - \tfrac{1}{2}gt^2 \qquad (2).$$

We can divide equation (2) by t and we get

$$v_0 \sin\theta = \tfrac{1}{2}gt.$$

We rewrite the first equation (1) as $t = \Delta x / v_0 \cos\theta$, which we substitute into the second equation to get $v_0 \sin\theta = \tfrac{1}{2}g[\Delta x / v_0 \cos\theta]$. Getting v_0 by itself, we have $v_0 = \{g\Delta x/(2\sin\theta \cos\theta)\}^{1/2}$. Plugging in the appropriate numbers, we get v_0 = 20.61 m/s = 74.2 km/h. This is the speed that the motorcyclist must have on liftoff to successfully reach the other ramp.

We can substitute this value into $t = \Delta x / v_0 \cos\theta$ to find the time in air to be 2.10 s.

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19. While preparing your cat's favourite meal, you drop the can and it rolls horizontally off the countertop and hits your cat on the head. Your cat's head is 0.95 m below the level of the countertop and 1.3 m away. Luckily, it was a can of *Tender Vittles* and the cat was not seriously hurt. What was the speed of the can as it left the countertop?

A 2D kinematics problem is solved by considering it to be two 1D kinematics problems. To do this we consider the ***x*** and ***y*** components separately.

x	***y***
$v_{0x} = v_0$	$v_{0y} = 0$ (horizontal flight)
$a_x = 0$	$a_y = -g = -9.81$ m/s^2
t = ?	t = ?
$\Delta x = +1.30$ m	$\Delta y = -0.95$ m (note sign!)
v_{fx} = ?	v_{fy} = ?

Considering the ***y*** information, we see that we can use the equation $\Delta y = v_0 t + \tfrac{1}{2}at^2$ to find the time in air. Using the given data,

$$-0.95 \; m = (0) + \tfrac{1}{2}(-9.81 \; m/s^2)(t)^2 .$$

From this we find that t = 0.4401 s.

We can use the equation $\Delta x = v_{0x}t + \tfrac{1}{2}a_x t^2$, to find the initial x component of velocity,

$$+1.3 \; m = v_{0x}t + \tfrac{1}{2}(0)(t)^2 .$$

thus

$$v_{0x} = 1.3\ m\ /\ 0.4401\ s = 2.95\ m/s\ .$$

the can leaves the countertop at 2.95 m/s.

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